

CONCURSUL NAȚIONAL DE MATEMATICĂ

“TEHNICI MATEMATICE“- ediția a XVIII-a

Etapa județeană 10.02.2023

Barem de corectare

Clasa a X-a – Matematică *M_tehnologic*

Subiectul I (30 p)

a) $2023 = 7 \cdot 17^2$ 4 p

$D_{2023} = \{1, 7, 17, 119, 289, 2023\}$4 p

$S = 1 + 7 + 17 + 119 + 289 + 2023 = 2456$2 p

b) $p = 2^{1+3+5+\dots+2023} = 2^{1012^2}$ 5 p

$p = (2^{1012 \cdot 506})^2 = pp$5 p

c) $x + a \geq 0 \Rightarrow x \geq -a$2 p

not. $t = \sqrt[4]{x+a} \Rightarrow t \geq 0$2 p

$t^2 + t - 2 = 0 \Rightarrow t \in \{-2, 1\}$2 p

$t \geq 0 \Rightarrow t = 1 \Rightarrow \sqrt[4]{x+a} = 1 \Rightarrow x + a = 1 \Rightarrow x = 1 - a$2 p

$1 - a \geq -a \Rightarrow S = \{1 - a\}$2 p

Subiectul II (30 p)

a) $4^x - 2^{x+3} + 16 = (2^x)^2 - 2 \cdot 4 \cdot 2^x + 4^2$ 5 p

$4^x - 2^{x+3} + 16 = (2^x - 4)^2 \geq 0, (\forall)x \in \mathbb{R}$5 p

b) $a^2 + b^2 + c^2 + 1 = 2(a + b + c - 1) \Leftrightarrow (a - 1)^2 + (b - 1)^2 + (c - 1)^2 = 0$6 p

Cum $(a - 1)^2 \geq 0, (b - 1)^2 \geq 0, c^2 \geq 0 \Rightarrow a - 1 = b - 1 = c - 1 = 0$ 3 p

$a = b = c = 1$1 p

c) În relația de la b) înlocuim $a = 2^x, b = 3^x, c = 5^x$3 p

Obținem relația $(2^x)^2 + (3^x)^2 + (5^x)^2 + 1 = 2(2^x + 3^x + 5^x - 1) \Leftrightarrow$

$\Leftrightarrow 4^x + 9^x + 25^x = 2^{x+1} + 2(3^x + 5^x) - 3$3 p

De unde rezultă $2^x = 3^x = 5^x = 1$3 p

$x = 0$1 p

Subiectul III (30 p)

a) $S = (1 + i)(1 + i)^{2022} + (1 - i)(1 - i)^{2022}$ 2 p

$S = (1 + i)[(1 + i)^2]^{1011} + (1 - i)[(1 - i)^2]^{1011}$ 2 p

$S = (1 + i)(2i)^{1011} + (1 - i)(-2i)^{1011}$ 2 p

$S = 2^{1012} \cdot i^{1012}$ 2 p

$S = 2^{1012} \cdot (i^4)^{253} = 2^{1012}$ 2 p

b) $\left| \frac{i}{1-\sqrt{3}i} \cdot \frac{\sqrt{2}-i\sqrt{2}}{2\sqrt{2}-i} \cdot \frac{2+\sqrt{5}i}{2\sqrt{3}+2i} \cdot \frac{3-\sqrt{7}i}{4+3i} \cdot \frac{1-2\sqrt{6}i}{2\sqrt{5}+4i} \right| = \frac{|i|}{|1-\sqrt{3}i|} \cdot \frac{|\sqrt{2}-i\sqrt{2}|}{|2\sqrt{2}-i|} \cdot \frac{|2+\sqrt{5}i|}{|2\sqrt{3}+2i|} \cdot \frac{|3-\sqrt{7}i|}{|4+3i|} \cdot \frac{|1-2\sqrt{6}i|}{|2\sqrt{5}+4i|}$ 2 p

$\frac{|i|}{|1-\sqrt{3}i|} = \frac{1}{2}$ 1 p

$\frac{|\sqrt{2}-i\sqrt{2}|}{|2\sqrt{2}-i|} = \frac{2}{3}$ 1 p

$\frac{|2+\sqrt{5}i|}{|2\sqrt{3}+2i|} = \frac{3}{4}$ 1 p

$\frac{|3-\sqrt{7}i|}{|4+3i|} = \frac{4}{5}$ 1 p

$\frac{|1-2\sqrt{6}i|}{|2\sqrt{5}+4i|} = \frac{5}{6}$ 1 p

$\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5} \cdot \frac{5}{6} = \frac{1}{6}$ 3 p

c) not. $z^2 = t$ 1 p

$t^2 - 12it - 100 = 0 \Rightarrow t_1 = 6i + 8, t_2 = 6i - 8$ 4 p

$t_1 = 6i + 8 \Rightarrow z_{1,2} = \pm \sqrt{6i + 8} = \pm(3 + i)$ 2 p

$t_2 = 6i - 8 \Rightarrow z_{3,4} = \pm \sqrt{6i - 8} = \pm(1 + 3i)$ 2 p

$S = \{3 + i, 3 - i, 1 + 3i, 1 - 3i\}$ 1 p