CONCURSUL NAȚIONAL DE MATEMATICĂ

“TEHNICI MATEMATICE“- ediția a XVIII-a

Etapa județeană 10.02.2023

Barem de corectare

Clasa a X-a – Matematică *M_șt-nat*

Subiectul I (30 p)

a) $x_1 = x_2 = \cdots = x_{2023} = n, (\forall) n \geq 2, n \in \mathbb{N}$……..…2 p

$a = n^1 \cdot n^2 \cdot n^3 \cdot \ldots \cdot n^{2023}$.....2 p

$a = n^{1+2+3+\cdots+2023}$……..…2 p

$a = (n^{2023 \cdot 506})^2 = pp$……..…4 p

b) $sin^4 x + cos^4 x + \frac{1}{2} sin^2 2x = (sin^2 x + cos^2 x)^2 = 1$.....2 p

$x = 4 + 2 = 6$……..…2 p

$3sin^4 x + 3cos^4 x = 3 - 6sin^2 x \cdot cos^2 x$.....2 p

$2sin^6 x + 2cos^6 x = 2 - 6sin^2 x \cdot cos^2 x$.....2 p

$y = -3 + 10 = 7$……..…1 p

$x < y$……..……1 p

c) $1 + 2^{-1} + 2^{-2} + 2^{-3} + \cdots + 2^{-n} = \frac{\left(\frac{1}{2}\right)^{n+1} - 1}{\frac{1}{2} - 1}$.....5 p

$1 + 2^{-1} + 2^{-2} + 2^{-3} + \cdots + 2^{-n} = 2 - \frac{1}{2^n}$.....4 p

$1 + 2^{-1} + 2^{-2} + 2^{-3} + \cdots + 2^{-n} < 2$.....1 p

Subiectul II (30 p)

a) Se arată că $\left(1 - \sqrt{2}\right)^3 = 7 - 5\sqrt{2}$……5 p

$\sqrt[3]{7 - 5\sqrt{2}} = \sqrt[3]{\left(1 - \sqrt{2}\right)^3} = 1 - \sqrt{2}$.....5 p

b) Se arată că $\left(1 + \sqrt{2}\right)^3 = 7 + 5\sqrt{2}$……..…..3 p

$\sqrt[3]{7 + 5\sqrt{2}} = \sqrt[3]{\left(1 + \sqrt{2}\right)^3} = 1 + \sqrt{2}$.....2 p

$q = 1 + \sqrt{2} + \frac{1}{1+\sqrt{2}} = 1 + \sqrt{2} - (1 - \sqrt{2})$.....4 p

$q = 2\sqrt{2} \in \mathbb{R} \setminus \mathbb{Q}$……..……1 p

c) $\sqrt[6]{x^4 \cdot \sqrt[3]{x^2 \cdot \sqrt{x}}} = x^{\frac{29}{36}}$……..……3 p

$\sqrt[5]{x^3 \cdot \sqrt[4]{x^2 \cdot \sqrt[3]{x}}} = x^{\frac{43}{60}}$……..…3 p

$x^{\frac{4}{45}} = \sqrt{2}$……..2 p

$x = 32\sqrt[8]{32}$…….……..2 p

Subiectul III (30 p)

a) $z = x + yi, unde\ x, y \in \mathbb{R}$……..……2 p

$\left|z + \frac{1}{a}\right|^2 - \left|\bar{z} - \frac{1}{a}\right|^2 = \sqrt{\left(x + \frac{1}{a}\right)^2 + y^2}^2 - \sqrt{\left(x - \frac{1}{a}\right)^2 + (-y)^2}^2 = \frac{4x}{a}$…….…… 6 p

$\frac{a}{4}\left(\left|z + \frac{1}{a}\right|^2 - \left|\bar{z} - \frac{1}{a}\right|^2\right) = x = Re(z)$……..…..2 p

b) $(6 + 9i)^2 = -45 + 108i$

$(1 + 8i)^2 = -63 + 16i$

$2(6 + 9i)^2 - 3(1 + 8i)^2 = 99 + 168i$

$99 + 168i = a + bi \Longrightarrow a = 99, b = 168$

c)

$2(6 + 9i)^n - 3(1 + 8i)^n = 3(7 + 4i)^n \Leftrightarrow 2 \cdot 3^{n-1} \cdot (2 + 3i)^n = (1 + 8i)^n + (7 + 4i)^n$…2 p

$2 \cdot 3^{n-1} \cdot (2 + 3i)^n = (1 + 8i)^n + (7 + 4i)^n \Leftrightarrow 2 \cdot 3^{n-1} = \left(\frac{1+8i}{2+3i}\right)^n + \left(\frac{7+4i}{2+3i}\right)^n$ …….3 p

$2 \cdot 3^{n-1} = \left(\frac{1+8i}{2+3i}\right)^n + \left(\frac{7+4i}{2+3i}\right)^n \Leftrightarrow 2 \cdot 3^{n-1} = \left(\frac{26-13i}{13}\right)^n + \left(\frac{26+13i}{13}\right)^n$…….……3 p

$2 \cdot 3^{n-1} = \left(\frac{26-13i}{13}\right)^n + \left(\frac{26+13i}{13}\right)^n \Leftrightarrow 2 \cdot 3^{n-1} = (2 + i)^n + (2 - i)^n$……...2 p