CONCURSUL NAȚIONAL DE MATEMATICĂ
„TEHNICI MATEMATICE”-editia a XVIII-a
Etapa judeteană 10.02.2023
Clasa a XI -a Matematică *M_șt-nat*

Barem de corectare

Subiectul I (30p)

a) $B=2\begin{pmatrix}\frac{\sqrt{3}}{2} & -\frac{1}{2}\\ \frac{1}{2} & \frac{\sqrt{3}}{2}\end{pmatrix}=$ 4p

$2\begin{pmatrix}\cos\frac{\pi}{6} & -\sin\frac{\pi}{6}\\ \sin\frac{\pi}{6} & \cos\frac{\pi}{6}\end{pmatrix}=$ 4p

$2A\left(\frac{1}{6}\right)$ 3p

b) $B^{2022}=\left(2A\left(\frac{1}{6}\right)\right)^{2022}=2^{2022}A^{2022}\left(\frac{1}{6}\right)=$ ....3p.

$2^{2022}\begin{pmatrix}\cos\frac{2022\pi}{6} & -\sin\frac{2022\pi}{6}\\ \sin\frac{2022\pi}{6} & \cos\frac{2022\pi}{6}\end{pmatrix}=2^{2022}\begin{pmatrix}\cos 337\pi & -\sin 337\pi\\ \sin 337\pi & \cos 337\pi\end{pmatrix}=$ 4p

$2^{2022}\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}$ 3p

c) $\left(A(x)\right)^n=\begin{pmatrix}\cos n\pi x & -\sin n\pi x\\ \sin n\pi x & \cos n\pi x\end{pmatrix}$ 4p

$\left(A(x)\right)^n=I_2 \Leftrightarrow n\pi x=2k\pi \Leftrightarrow x=\frac{2k}{n}, k\in\mathbb{Z}$ 3p

$\Rightarrow x=\frac{2}{n}$ este cel mai mic număr real strict pozitiv cu $\left(A(x)\right)^n=I_2$ 3p

Subiectul II (30p)

a) $AA': y = x + 3$ 2p

Fie $d_1 \cap AA' = \{M\} \Rightarrow M\left(-\frac{3}{2}, \frac{3}{2}\right)$ ....2p.

M mijlocul lui $AA' \Rightarrow A'(-4, -1)$ 2p

$A_{\triangle OAA'} = \frac{1}{2}|\Delta|, \Delta = \begin{vmatrix} 0 & 0 & 1 \\ 1 & 4 & 1 \\ -4 & -1 & 0 \end{vmatrix} = 15$ 3p

$A_{\triangle OAA'} = \frac{15}{2}$ 1p

b) $dist(M, d_1) = \frac{|r+s|}{\sqrt{2}}$, ...5p

$dist(M, d_2) = \frac{|r-s|}{\sqrt{2}}$ 5p

c) $r^2 - s^2 = 1 \Rightarrow s^2 = r^2 - 1$ și cum $s < 0 \Rightarrow s = -\sqrt{r^2 - 1}$ 3p

$dist(M, d_1) = \frac{\left|r - \sqrt{r^2 - 1}\right|}{\sqrt{2}}, dist(M, d_2) = \frac{\left|r + \sqrt{r^2 - 1}\right|}{\sqrt{2}}$ 1p

$\lim\limits_{r \to \infty} dist(M, d_1) = \lim\limits_{r \to \infty} \frac{\left|r - \sqrt{r^2 - 1}\right|}{\sqrt{2}} = 0$ 2p

$\lim\limits_{r \to -\infty} dist(M, d_1) = \infty$ 1p

$\lim\limits_{r \to \infty} dist(M, d_2) = \lim\limits_{r \to \infty} \frac{\left|r + \sqrt{r^2 - 1}\right|}{\sqrt{2}} = \infty$ 1p

$\lim\limits_{r \to -\infty} dist(M, d_2) = \lim\limits_{r \to \infty} \frac{\left|-r + \sqrt{r^2 - 1}\right|}{\sqrt{2}} = 0$ 2p

Subiectul III (30p)

a) $\lim_{x\to\infty}\sqrt{x^2+x+1}=\infty \Rightarrow$ nu avem asimptoptă orizontală 2p

$m=\lim_{x\to\infty}\frac{\sqrt{x^2+x+1}}{x}=\lim_{x\to\infty}\frac{x\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}}{x}=\lim_{x\to\infty}\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}=1$ 2p

$n=\lim_{x\to\infty}\left(\sqrt{x^2+x+1}-x\right)=\lim_{x\to\infty}\frac{x+1}{\sqrt{x^2+x+1}+x}=$ 2p

$\lim_{x\to\infty}\frac{x\left(1+\frac{1}{x}\right)}{x\left(\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+1\right)}=\frac{1}{2}$ 2p

$y=x+\frac{1}{2}$ este asimptotă oblică spre $+\infty$ 2p

b) $\lim_{x\to 1}\frac{\sin\left(ax^2+bx+c\right)}{x^3-x}=\lim_{x\to 1}\frac{\sin\left[\pi-\left(ax^2+bx+c\right)\right]}{x^3-x}=$ 3p

$\lim_{x\to 1}\frac{\sin\left[\pi-\left(ax^2+bx+c\right)\right]}{\pi-\left(ax^2+bx+c\right)}\cdot\frac{\pi-\left(ax^2+bx+c\right)}{x^3-x}=\lim_{x\to 1}\frac{\pi-\left(ax^2+bx+c\right)}{x^3-x}=$ 3p

Inlocuind $c=\pi-a-b$ 1p

Obtinem $\lim_{x\to 1}\frac{(x-1)\left[-a(x+1)-b\right]}{x(x-1)(x+1)}=\frac{-2a-b}{2}$ ....3p.

c) $\lim_{x\to\infty}xg(x)\left[x-\sqrt{x^2+x+1}\cdot\frac{\ln\left(e^x+x\right)}{x}\right]=$ 1p

$\lim_{x\to\infty}\frac{\sin\left(ax^2+bx+c\right)}{x^2-1}\left[x-\sqrt{x^2+x+1}+\sqrt{x^2+x+1}-\sqrt{x^2+x+1}\cdot\frac{\ln\left(e^x+x\right)}{x}\right]$ 3p

Avem,

$\lim_{x\to\infty}\frac{\sin\left(ax^2+bx+c\right)}{x^2-1}=0$ 1p

$\lim_{x\to\infty}\left(x-\sqrt{x^2+x+1}\right)+\lim_{x\to\infty}\frac{\sqrt{x^2+x+1}}{x}\left(x-\ln\left(e^x+1\right)\right)=$ 1p

$-\frac{1}{2}+1\cdot\lim_{x\to\infty}\left(x-\ln\left(e^x+1\right)\right)=-\frac{1}{2}+\lim_{x\to\infty}\ln\frac{1}{1+\frac{1}{e^x}}=-\frac{1}{2}+0=-\frac{1}{2}$ 3p

Deci limita este $0\cdot\left(-\frac{1}{2}\right)=0$. 1p