CONCURSUL NAŢIONAL DE MATEMATICĂ

„TEHNICI MATEMATICE”-editia a XVIII-a

Etapa judeteană 10.02.2023

Clasa a XII -a *M_Tehnologic*

SUBIECTUL I

1)	$a_6 = a_1 + 5r,\ a_2 = a_1 + r \Rightarrow\ a_6 = a_2 + 4r$ $16 = 8 + 4r \Rightarrow r = 2$	3p 2p
2)	$2x + 1 \geq 3 \cdot 3 + 2$ $2x \geq 10 \Rightarrow x \in [5, +\infty)$	2p 3p
3)	$2^{6x} = 2^4$ $6x = 4 \Rightarrow x = \frac{2}{3}$	3p 2p
4)	$C_{2023}^{\ 2} = \frac{2023!}{2021!\,2!} = \frac{2022 \cdot 2023}{2}$ $A_{2023}^{\ 2} = \frac{2023!}{2021!} = 2022 \cdot 2023$ Deci $\frac{C_{2023}^{\ 2}}{A_{2023}^{\ 2}} = \frac{1}{2}$	2p 2p 1p
5)	$AB = \sqrt{(-1-0)^2 + (3-1)^2} = \sqrt{5}, AC = \sqrt{(-1-2)^2 + (3+2)^2} = \sqrt{34},$ $BC = \sqrt{(2-0)^2 + (2+1)^2} = \sqrt{13}$ $P_{\Delta ABC} = AB + AC + BC = \sqrt{5} + \sqrt{34} + \sqrt{13}$	3p 2p
6)	$sin30° = \frac{1}{2}, ctg45° = 1, cos60° = \frac{1}{2}$ $\Rightarrow E = 1$	3p 2p
SUBIECTUL II		
1)	a) $detA = \begin{vmatrix} 5 & 1 \\ -2 & -1 \end{vmatrix} = -5 + 2$ $= -3$	3p 2p
	b) $A \cdot A = \begin{pmatrix} 5 & 1 \\ -2 & -1 \end{pmatrix} \cdot \begin{pmatrix} 5 & 1 \\ -2 & -1 \end{pmatrix} = \begin{pmatrix} 23 & 4 \\ -8 & -1 \end{pmatrix} \Rightarrow$ $A^2 - 23\mathrm{I}_2 = \begin{pmatrix} 23 & 4 \\ -8 & -1 \end{pmatrix} - \begin{pmatrix} 23 & 0 \\ 0 & 23 \end{pmatrix} = \begin{pmatrix} 0 & 4 \\ -8 & -24 \end{pmatrix}$	3p 2p
	c) $A \cdot \mathrm{X} - \mathrm{I}_2 = A^2 - 23\mathrm{I}_2 \Rightarrow A \cdot \mathrm{X} = \mathrm{I}_2 + A^2 - 23\mathrm{I}_2 \Rightarrow A \cdot \mathrm{X} = \begin{pmatrix} 1 & 4 \\ -8 & -23 \end{pmatrix}, detA = -3 \neq 0$	3p

	$\Rightarrow$ A inversabilă$\Rightarrow A^{-1}=\frac{1}{detA}A^{*}=\frac{1}{3}\begin{pmatrix}1 & 1\\ -2 & -5\end{pmatrix}$ deci $X=\frac{1}{3}\begin{pmatrix}1 & 1\\ -2 & -5\end{pmatrix}\begin{pmatrix}1 & 4\\ -8 & -23\end{pmatrix}\Rightarrow X=\frac{1}{3}\begin{pmatrix}-7 & -19\\ 38 & 107\end{pmatrix}$	2p
2)	a) $3\circ 2023=3\cdot 2023-3\cdot 3-3\cdot 2023+12=$ $=3$	3p 2p
	b) $x\circ y=xy-3x-3y+12=xy-3x-3y+9+3=x(y-3)-3(y-3)+3$ Deci $x\circ y=(x-3)(y-3)+3$	3p 2p
	c) $a\circ b=13\Rightarrow (a-3)(b-3)+3=13\Rightarrow (a-3)(b-3)=10$, $a,b\in \boldsymbol{N}$, deci $((a-3),(b-3))\in\{(1,10),(2,5),(5,2),(10,1)\}$ $(a,b)\in\{(4,13),(5,8),(8,5),(13,4)\}$	2p 3p
SUBIECTUL III		
1)	a) f *este funcție derivabilă pe* $\boldsymbol{R}$, $f'(x)=\left((x^2+4)e^x\right)'=$ $=(x^2+4)'e^x+(x^2+4)(e^x)'=(2x+x^2+4)e^x$	3p 2p
	b) $\lim\limits_{x\to\infty}\frac{f'(x)}{f(x)}=\lim\limits_{x\to\infty}\frac{(x^2+2x+4)e^x}{(x^2+4)e^x}=\left(\frac{\infty}{\infty}\right)=$ $=\lim\limits_{x\to\infty}\frac{x^2\left(1+\frac{2}{x}+\frac{4}{x^2}\right)}{x^2\left(1+\frac{4}{x^2}\right)}=1$	3p 2p
	c) $f'(x)=0\Rightarrow$ ecuația nu are soluții reale Obținem $f'(x)>0$ pentru orice $x\in\boldsymbol{R}$, deci f strict crescătoare pentru $x\in\boldsymbol{R}$, $\lim\limits_{x\to-\infty}f(x)=\lim\limits_{x\to+\infty}\frac{x^2+4}{e^x}=0$, iar $\lim\limits_{x\to+\infty}f(x)=\lim\limits_{x\to+\infty}(x^2+4)e^x=+\infty$ și $f(0)=4$ Deci $f(x)\geq 4$, pentru orice număr real $x\geq 0$.	3p 2p
2)	a) $\int_2^3 f(x)\cdot(x+2)dx=\int_2^3\frac{x-2}{x+2}\cdot(x+2)dx=\int_2^3(x-2)dx=$ $=\left(\frac{x^2}{2}-2x\right)\Big\vert_2^3=\left(\frac{3^2}{2}-6\right)-\left(\frac{2^2}{2}-4\right)=\frac{1}{2}$	3p 2p
	b) $\int_1^2 f(x)dx=\int_1^2\frac{x-2}{x+2}dx=\int_1^2(1-\frac{4}{x+2})dx=(x-4\ln(x+2))\Big\vert_1^2=$ $=(2-4\ln 4))-(1-4\ln 3)=1-4ln\frac{4}{3}$	3p 2p

	$c)\ \int_2^m f(x)\,f'(x)dx = \frac{f^2(x)}{2}\Big|_2^m = \frac{1}{2}\left(\frac{m-2}{m+2}\right)^2$ Obținem $\frac{1}{2}\left(\frac{m-2}{m+2}\right)^2 = \frac{1}{18} \Rightarrow \left(\frac{m-2}{m+2}\right)^2 = \frac{1}{9} \Rightarrow \frac{m-2}{m+2} = \pm\frac{1}{3}$ Deoarece $m > 2$, găsim $m = 4$, care convine.	2p 3p